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B.Sc. – III (Semester – VI) (CGPA) Examination, 2018
MATHEMATICS (Special Paper – XIII)
Integral Transform

Day and Date : Tuesday, 03-04-2018

Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions: 1) All questions are compulsory.
 2) Figures to the right indicate full marks.

1. Choose the correct alternative for each of the following : 14

1) $L\{e^{3t} t^2\} =$

- a) $\frac{1}{(P-3)^2}$ b) $\frac{2}{(P-3)^3}$ c) $\frac{3}{(P-3)^2}$ d) $\frac{2}{(P-3)^2}$

2) $L\left\{\frac{\cos at}{t}\right\} =$

- a) $\tan^{-1}\left(\frac{a}{P}\right)$ b) $\frac{1}{a} \tan^{-1}\left(\frac{a}{P}\right)$ c) $\tan^{-1}\left(\frac{P}{a}\right)$ d) None of these

3) $L^{-1}\left\{\frac{1}{P^{n+1}}\right\} =$

- a) $\frac{t^{n+1}}{\lceil n+1 \rceil}$ b) $\frac{t^n}{\lceil n+1 \rceil}$ c) $\frac{t^{n-1}}{\lceil n+1 \rceil}$ d) $\frac{t^{n+1}}{\lceil n \rceil}$

4) $L\{F(t)\} = f(P)$ then $L\{e^{-at} F(t)\} =$

- a) $\frac{1}{a} f(P/a)$ b) $f\left(\frac{P}{a}\right)$ c) $f(P+a)$ d) $f(P-a)$

5) $L^{-1}\left\{\frac{1}{(P+a)^n}\right\} =$

- a) $\frac{e^{-at} t^{n-1}}{(n-1)!}$ b) $\frac{e^{at} t^{n-1}}{(n-1)!}$ c) $\frac{e^{-at} t^n}{n!}$ d) $\frac{e^{-at} t^{n+1}}{\lceil n+1 \rceil}$

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6) If $L^{-1}\{f(P)\} = F(t)$ then $L^{-1}\left\{\int_P^{\infty} f(x)dx\right\} =$

- a) $F'(t)$ b) $-tF'(t)$ c) $tF(t)$ d) $\frac{F(t)}{t}$

7) $L\{\cosh at\} =$

- | | |
|--------------------------|--------------------------|
| a) $\frac{1}{P^2 - a^2}$ | b) $\frac{a}{P^2 - a^2}$ |
| c) $\frac{P}{P^2 - a^2}$ | d) $\frac{P}{P^2 + a^2}$ |

8) If $L\{F(t)\} = f(P)$ then $L\{F''(t)\} =$

- | | |
|------------------------------|------------------------------|
| a) $Pf(P) - F(0) - F'(0)$ | b) $P^2f(P) - PF(0) - F'(0)$ |
| c) $P^2f(P) - PF'(0) - F(0)$ | d) $P^2f(P) - F(0) - PF'(0)$ |

9) $L\{tsin at\} =$

- | | | | |
|-----------------------------|------------------------------|---------------------------------|--------------------------------|
| a) $\frac{2a}{(P^2 + a^2)}$ | b) $\frac{2aP}{(P^2 + a^2)}$ | c) $\frac{-2aP}{(P^2 + a^2)^2}$ | d) $\frac{2aP}{(P^2 + a^2)^2}$ |
|-----------------------------|------------------------------|---------------------------------|--------------------------------|

10) $\int_0^{\infty} \frac{\sin t}{t} dt =$

- a) $\frac{\pi}{2}$ b) π c) 2π d) 3π

11) $L\left\{\frac{1}{a}(e^{at} - 1)\right\} =$

- | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|
| a) $\frac{1}{P(P-a)}$ | b) $\frac{a}{P(P-a)}$ | c) $\frac{aP}{(P-a)}$ | d) $\frac{P}{a(P-a)}$ |
|-----------------------|-----------------------|-----------------------|-----------------------|

12) $1 * 1 * 1 * \dots * 1$ (n times) =

- | | | | |
|--------------------|---------------------|-----------------------------|--------------------------|
| a) $\frac{t^n}{n}$ | b) $\frac{t^n}{n!}$ | c) $\frac{t^{n-1}}{(n-1)!}$ | d) $\frac{t^{n-1}}{n-1}$ |
|--------------------|---------------------|-----------------------------|--------------------------|



13) If $L^{-1}\{f(P)\} = F(t)$ then $L^{-1}\{f(aP)\} =$

a) $F\left(\frac{t}{a}\right)$ b) $\frac{1}{a}F\left(\frac{t}{a}\right)$

c) $\frac{1}{a}F\left(\frac{a}{t}\right)$ d) $aF(t)$

14) $L^{-1}\left\{\frac{1}{\sqrt{P}}\right\} =$

a) $\frac{1}{\sqrt{\pi}}$ b) $\frac{1}{\sqrt{\pi t}}$ c) $\frac{\pi}{\sqrt{t}}$ d) $\frac{1}{\pi t}$

2. Attempt **any seven** of the following :

14

1) Using the definition of the Laplace Transform find the Laplace Transform of $F(t)$ where

$$\begin{aligned} F(t) &= 3 & 0 < t < 5 \\ &= 0 & t > 5 \end{aligned}$$

2) Find the value of $\int_0^{\infty} e^{-3t} t^s dt$.

3) Find $L^{-1}\left\{\frac{P-2}{(P-2)^2 + 5^2} + \frac{P+4}{(P+4)^2 + 9^2} + \frac{1}{(P+2)^2 + 3^2}\right\}$.

4) Find $L^{-1}\left\{\frac{1}{(P-1)(P-2)}\right\}$.

5) Find $L\left\{\frac{\sin t}{t}\right\}$.

6) Show that $L^{-1}\left\{\log\left(\frac{P+2}{P+1}\right)\right\} = \frac{1}{t}(e^{-t} - e^{-2t})$.

7) If $L^{-1}\{f(P)\} = F(t)$ and $F(0) = 0$ then $L^{-1}\{Pf(P)\} = F'(t)$.

8) Solve $\frac{d^2y}{dt^2} + y = 0$ under the conditions that $y = 1, \frac{dy}{dt} = 0$ when $t = 0$.

9) If $y(x, t)$ is a function of x and t prove that $L\left\{\frac{\partial y}{\partial t}\right\} = P\bar{y}(x, P) - y(x, 0)$.

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3. A) Attempt **any two** of the following :**10**

1) Find $L\{t(3\sin 2t - 2\cos 2t)\}$.

2) Find $L^{-1}\left\{\frac{1}{P^3(P^2 + 1)}\right\}$.

3) If $L\{F(t)\} = f(P)$ then $L\left\{\frac{F(t)}{t}\right\} = \int_P^\infty f(x)dx$.

B) If $y(x, t)$ is a function of x and t prove that**4**

i) $L\left\{\frac{\partial y}{\partial x}\right\} = \frac{dy}{dx}$

ii) $L\left\{\frac{\partial^2 y}{\partial x^2}\right\} = \frac{d^2y}{dx^2}$

4. Attempt **any two** of the following :**14**1) Let $F(P)$ and $G(P)$ be two polynomials in P where $F(P)$ has degree less than that of $G(P)$. If $G(P)$ has n distinct zeros α_k , $k = 1, 2, \dots, n$, i.e.

$$G(P) = (P - \alpha_1)(P - \alpha_2) \dots (P - \alpha_n) \text{ then } L^{-1}\left\{\frac{F(P)}{G(P)}\right\} = \sum \frac{P(\alpha_k)}{Q'(\alpha_k)} e^{\alpha_k t}.$$

2) Illustrate the initial-value theorem and final-value theorem for the function $F(t) = 3e^{-2t}$.3) Solve $(D - 2)x + 3y = 0$ $2x + (D - 1)y = 0$, if $x(0) = 8$ and $y(0) = 3$.5. Attempt **any two** of the following :**14**1) If $L^{-1}\{f(P)\} = F(t)$ and $L^{-1}\{g(P)\} = G(t)$ then

$$L^{-1}\{f(P).g(P)\} = \int_0^t F(u)G(t-u)du = F * G$$

2) Solve the following differential equation using Laplace transforms

$$Y'' + Y = 2, Y(0) = 3, Y'(0) = 1.$$

3) If $F(t) = t^2$, $0 < t < 2$ and $F(t+2) = F(t)$. Find $L\{F(t)\}$.