SLR-ST – 188

Seat No.	

Set **P**

B.Sc. – III (Semester – V) (CGPA) Examination, 2018
STATISTICS (Special Paper – VII)
Statistical Inference – I

Day and Date : Friday, 13-4-2018 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : 1) ***All*** *questions are* ***compulsory****.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. Choose the correct alternative : **14**

1) Parameter is a
a) Sample characteristic b) Population characteristic
c) Part of a population d) Part of a sample

2) Efficiency of an estimator is related to
a) Mean b) Median c) Mode d) Variance

3) If T is unbiased for θ then $\varnothing(T)$ is unbiased for $\varnothing(\theta)$ if $\varnothing$ is
a) Linear b) Continuous c) Onto d) One-to-one

4) Sample variance is ____________ estimator of population variance.
a) Unbiased b) Consistent
c) Both a) and b) d) None of these

5) Which of the following is always true ?
a) An unbiased estimator is unique
b) MLE is a function of sufficient statistics
c) Unbiased estimator is a consistent statistic
d) MLE always exists

6) The information function for the parameter θ of Poisson distribution is
a) θ b) $1/\theta$ c) θ^2 d) None of these

7) The denominator in the Cramer-Rao inequality is known as
a) Information unit b) Lower bound of variance
c) Upper bound of variance d) All of these

8) Based on a random sample of size n, moment estimator of μ in $N(\mu, \sigma^2)$ is

a) $\sum x_i^2 / n$
b) $\sum x_i^2 /(n-1)$
c) $\sum x_i$
d) $\sum x_i / n$

9) If x is a single observation from L(0, λ) then the sufficient statistic for λ is

a) |x| b) x c) x^2 d) None of these

10) If T be an consistent estimator of θ then g(T) is consistent for g(θ) =

a) $2\theta + 3$ b) e^{θ} c) $\sqrt{\theta}$ d) All of these

11) By the method of moments one can estimate

a) All constants of a population
b) Only mean and variance of the distribution
c) All moments of a population distribution
d) All the above

12) If the variance of an estimator attains the Cramer-Rao lower bound, the estimator is

a) Most efficient
b) Sufficient
c) Unbiased
d) Admissible

13) If T_n and T^*_n are two unbiased estimator such that $v(T_n) = v(T^*_n)$ then

a) T_n and T^*_n may not be same
b) $T_n = T^*_n$
c) T_n is more efficient
d) None of these

14) Likelihood function is a function of

a) Sample only
b) Parameter only
c) Either sample or parameter
d) Sample and parameter

2. Attempt **any seven** of the following : **14**

1) Define a statistic and give one example.

2) Show that sample mean is unbiased estimator of population mean.

3) Let $T_1 = \frac{x_1 + x_2}{2}$, $T_2 = \frac{2x_1 + 3x_2}{5}$ are two unbiased estimators of population mean. Then find the efficiency of T_2 with respect to T_1.

4) Define consistent estimator.

5) Give one example of biased but consistent estimator.

6) Obtain likelihood function of a random sample $x_1, x_2, \ldots\ldots x_n$ from geometric distribution with parameter p.

7) State Neyman-factorization theorem.

8) Define Minimum Variance Unbiased Estimator (MVUE).

9) Show that if T is unbiased estimator of θ then T^2 is not unbiased estimator of θ^2.

3. A) Attempt **any two** from the following : **10**

1) Find moment estimator of θ if $f(x) = (1 + \theta)\, x^{\theta}$ $\quad 0 < x < 1$

$\quad\quad 0 \quad$ otherwise

2) If L is the likelihood function of random sample $x_1, x_2, \ldots\ldots x_n$ from $f(x, \theta)$. Show that $E\left(\frac{\delta \log L}{\delta\theta}\right)^2 = -E\left(\frac{\delta^2 \log L}{\delta\theta^2}\right)$.

3) Show that there exists infinite number of unbiased estimators of parameter θ.

B) Find information function for the parameter θ of Poisson distribution. **4**

4. Attempt **any two** of the following : **14**

1) Find MLE and Moment estimator of θ based on a sample of size n from

$$f(x, \theta) = 1 \qquad \theta - \frac{1}{2} < x < \theta + \frac{1}{2}$$
$$0 \qquad \text{otherwise}$$

2) Prove that a biased estimator is consistent if its bias and variance both tend to zero as the sample size tends to infinity.

3) Explain the concept of sufficiency. Let $x_1, x_2, \ldots\ldots x_n$ be a random sample of size n from a distribution with pdf

$$f(x, \theta) = \theta x^{\theta-1} \qquad 0 < x < 1$$
$$0 \qquad \text{Otherwise}$$

Obtain a sufficient statistic for θ.

5. Attempt **any two** of the following : **14**

1) A random sample $x_1, x_2, ... x_n$ is drawn from a normal population with unknown mean μ and known variance σ^2. The following are estimators of μ.

i) $T_1 = \frac{x_1 + x_2 + x_3 + x_4}{4}$

ii) $T_2 = \frac{x_1 + x_2}{2} + x_3$ and

iii) $T_3 = \frac{2x_1 + x_2 + \lambda x_3}{3}$.

Find λ where T_3 is unbiased estimator of μ. Are T_1 and T_2 unbiased ? State the estimator which is more efficient.

2) State and prove Cramer-Rao inequality.

3) Let $x_1, x_2, ... x_n$ denote the random sample from exponential distribution with pdf

$$f(x, \theta) = \theta e^{-\theta x} \quad x > 0, \ \theta > 0$$
$$0 \qquad \text{otherwise}$$

a) Obtain expression for likelihood function

b) Find Fisher information $I(\theta)$.