

Seat No.	
----------	--

B.Sc. (Semester - III) (CBCS) Examination Oct/Nov-2019
Mathematics (Paper - V)
DIFFERENTIAL CALCULUS

Day & Date: Monday, 14-10-2019
 Time: 03:00 PM To 05:30 PM

Max. Marks: 70

Instructions: 1) All questions are compulsory.
 2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. 14

- 1) The polar sub tangent equal to _____.
 - a) $\frac{d\theta}{dr}$
 - b) $r \frac{d\theta}{dr}$
 - c) $r^2 \frac{d\theta}{dr}$
 - d) $\frac{1}{r} \frac{d\theta}{dr}$
- 2) Angel of intersection of curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$ is _____.
 - a) π
 - b) $\frac{\pi}{2}$
 - c) 0
 - d) $-\frac{\pi}{2}$
- 3) The angle of intersection of two curves is defined as the angle between their _____.
 - a) normals
 - b) radius vectors
 - c) tangents
 - d) none of these
- 4) Pedal equation of a curve $r = e^{a \cot \alpha}$ is _____.
 - a) $p = r \sin \alpha$
 - b) $p = r \cos \alpha$
 - c) $p = r$
 - d) None of these
- 5) The intrinsic formula for radius of curvature is _____.
 - a) $\rho = \frac{dy}{dx}$
 - b) $\rho = \frac{ds}{d\psi}$
 - c) $\rho = \frac{1}{s} \frac{ds}{d\psi}$
 - d) none of these
- 6) Radius of curvature for $y = e^x$ at the point (0,1) is _____.
 - a) $2\sqrt{2}$
 - b) 0
 - c) $3\sqrt{2}$
 - d) none of these
- 7) The radius of curvature for $xy = c^2$ at any point is _____.
 - a) $\frac{(x^2+y^2)^{2/3}}{2c^2}$
 - b) $\frac{(x^2+y^2)^{3/2}}{2c}$
 - c) $\frac{(x^2+y^2)^{3/2}}{2c^2}$
 - d) none of these
- 8) The radius of curvature at any point on the hyperbola $pr = a^2$ is _____.
 - a) $\frac{r^3}{a^2}$
 - b) $\frac{r^2}{a^2}$
 - c) $\frac{r}{a^2}$
 - d) none of these
- 9) If $x = r \cos \theta$, $y = r \sin \theta$ the $\frac{\partial(r,\theta)}{\partial(x,y)} =$ _____.
 - a) r
 - b) $\frac{1}{r}$
 - c) a
 - d) None of these

- 10) If $u = x^2 - y^2, v = xy$, then $\frac{\partial(u,v)}{\partial(x,y)} =$ _____.
- a) $x^2 + y^2$ b) $2(x^2 + y^2)$
c) $x + y$ d) $2(x + y)$
- 11) If $u = x^2, v = y^2$ then $\frac{\partial(u,v)}{\partial(x,y)} =$ _____.
- a) xy b) $4xy$
c) $\frac{1}{xy}$ d) $\frac{1}{4xy}$
- 12) A function $f(x, y)$ is minimum at (a, b) if _____.
- a) $AC - B^2 > 0$ and $A > 0$ b) $AC - B^2 > 0$ and $A < 0$
c) $AC - B^2 < 0$ and $A > 0$ d) none of these
- 13) $f(x) = \sin x(1 + \cos x)$ is maximum at _____.
- a) $x = \pi$ b) $x = \pi/2$
c) $x = \pi/3$ d) none of these
- 14) The function $f(x) = x^3 - 6x^2 + 24x + 4$ has _____.
- a) minimum at $x = 6$ and maximum at $x = 4$
b) minimum at $x = 2$
c) maximum at $x = 2$
d) Neither maximum nor minimum at any point.

Q.2 A) Answer the following questions. (Any Four)

08

- 1) Find the equation of a tangent at any point of $y^2 = 4ax$.
- 2) Find the angle between $y = \sin x$ and $y = \cos x$.
- 3) Find the radius of curvature at any point for $y = c \cdot \log(\sec \psi)$.
- 4) Find $\frac{ds}{dt}$ for $x = a(t - \sin t), y = a(1 - \cos t)$
- 5) State the necessary condition for the function of one variable to a maximum value.

B) Answer the following questions. (Any Two)

06

- 1) If $u = \frac{yz}{x}, v = \frac{xz}{y}, \omega = \frac{xy}{z}$ then find $\frac{\partial(u,v,\omega)}{\partial(x,y,z)}$
- 2) Find the maximum and minimum value of the polynomial $f(x) = 2x^3 - 15x^2 + 36x + 10$.
- 3) Find the radius of curvature for $r = 3(1 + \cos\theta)$ at any point on it.

Q.3 A) Answer the following questions. (Any Two)

08

- 1) Obtain the expression for radius of curvature for the curve given in parametric form $x = \phi(t), y = \psi(t)$.
- 2) Find the equation of a tangent and Normal at any point on it to the circle $x^2 + y^2 = a^2$.
- 3) Find the extreme values of xy^2z^3 subject to the condition $x + y + z = 6$

B) Answer the following questions. (Any One)

06

- 1) If ρ_1 and ρ_2 are radii of curvature at the extremities of a focal chord of a parabola $y^2 = 4ax$ then prove that $\rho_1^{-2/3} + \rho_2^{-2/3} = (2a)^{-2/3}$
- 2) If J be a Jacobian of u, v, w with respect to x, y, z and J' be a Jacobian of x, y, z with respect to u, v, w then prove that $J.J' = 1$

- Q.4 A) Answer the following questions. (Any Two) 10**
- 1) Find the radius of curvature for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point on it.
 - 2) Obtain the expression for length of tangent, normal, sub tangent and subnormal, for the cartesian curve $y = f(x)$
 - 3) If $u^3 + v + w = x + y^2 + z^2$, $u + v^2 + w = x^2 + y + z^2$,
 $u + v + w^3 = x^2 + y^2 + z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
- B) Answer the following questions. (Any One) 04**
- 1) Determine the points where the function $x^3 + y^3 - 3axy$ has maximum and minimum value.
 - 2) Prove that the radius of a curvature of a circle is constant and it is equal to radius of circle.
- Q.5 Answer the following questions. (Any Two) 14**
- a) Define the angle of intersection of two curves. If θ is an angle between two curves then obtain the formula to find it and hence find angle of intersection of $y = 4 - x^2$ and $y = x^2$
 - b) Explain the Lagrange's method of undetermined multipliers to determine the extreme values of $u = f(x, y, z)$ subject to $\phi_1(x, y, z) = 0$ and $\phi_2(x, y, z) = 0$
 - c) Find the radius of curvature at any point (r, θ) on the curve $r^m = a^m \cdot \cos m\theta$ and show that ρ at any point to $r^2 = a^2 \cos 2\theta$ is $a^2/3r$