SLR-DK-34

Seat No.	

Set P

B.Sc. (Semester - I) (Old) (CBCS) Examination Oct/Nov-2019
Mathematics (Paper – II)
CALCULUS

Day & Date: Monday, 18-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) $\lim_{x\to 0} \log_x \sin x =$ ______.
   a) 0 b) -1
   c) 1 d) 2

2) $\lim_{x\to\infty} x^2 e^{-x} =$ ______.
   a) 0 b) 3
   c) $\frac{3}{2}$ d) e

3) $\lim_{x\to \pi/2} \frac{\tan x}{\tan 3x} =$ _______.
   a) $\frac{1}{2}$ b) 11
   c) -2 d) 3

4) The expansion of $\log(1-x)$ is ______.
   a) $-\left[x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots\right]$ b) $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$
   c) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$ d) $1-x+\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}-\cdots$

5) If $Y=(3x+2)^9$ then Y_{10} = ______.
   a) $9!\,3^9$ b) $\frac{9!}{10!}3^{10}(3x+2)^{10}$
   c) $\frac{9!}{1!}3^{10}(3x+2)^0$ d) 0

6) If $f(x,y)=\frac{x^3-y^3}{x^2+y^2},\ \ x^2+y^2 \neq 0\ and\ f(0,0)=0$ then $f_4\ (0,0) =$ _______.
   a) 1 b) 2
   c) -1 d) does not exist

7) If $f(x,y)=|x|+|y|$ then ______.
   a) f is not continuous at (0,0)
   b) f is continuous and differentiable at (0,0)
   c) f is continuous but not differentiable at (0,0)
   d) f is neither continuous nor differentiable at (0,0)

8) If $f(x,y)$ is a Homogenous function of degree ‘n’ then
   $\left\{\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)\ldots m\ times\right\} f(x,y) =$ ________.
   a) $n^m f(x,y)$ b) $m^n f(x,y)$
   c) $(n+m)\,f(x,y)$ d) $n(n-1)\,f(x,y)$

9) If $u = f\left(\frac{x}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ________.

a) 2 b) -1
c) $\frac{1}{2}$ d) 0

10) $\int_0^{\pi/2} \sin^4 x \cos^5 x \; dx$ = ______.

a) $\frac{2}{35}$ b) $\frac{3}{335}$
c) $\frac{8}{315}$ d) $\frac{2}{815}$

11) $\int_0^{\infty} \frac{dx}{(1+x^2)^5} =$ ______.

a) $\frac{5\pi}{32}$ b) $\frac{35}{256}\pi$
c) $\frac{16}{35}$ d) $\frac{8\pi}{35}$

12) If $\bar{a}$ is a constant vector and r and $\bar{r}$ have usual meanings, then $\nabla(\bar{a}.\bar{r})$= ______.

a) $-\frac{\bar{a}}{r}$ b) $\frac{\bar{a}}{r^3}$
c) $\bar{a}$ d) r

13) If $\bar{f} = x^2zi - 2y^3z^2j + xy^2zk$ then curl $\bar{f}$ at (1,-1, 1) is ______.

a) 0 b) 7
c) $-6i$ d) $8i$

14) The directional derivative of a scalar point function ϕ is maximum in the direction of _____.

a) $\nabla\phi$ b) $\nabla.\phi$
c) $\nabla \times \phi$ d) Curl grad ϕ

Q.2 A) Attempt any four of the following question. 08

1) Find y_5 of $y = \frac{\log x}{x}$.
2) Evaluate $\lim_{x \to 0} \frac{3^x - 2^x}{x}$.
3) Define the term limit of a two variables.
4) Evaluate $\int_0^{\pi/2} \sin^8 x \cos^4 x \, dx$.
5) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\bar{r}.\nabla\phi$

B) Attempt any two of the following questions. 06

1) State Taylor's and Maclaurin's series.
2) Examine for continuity at (0,0) the function $f(x,y) = \frac{x^3+y^3}{x-y}$ if $(x,y) \neq (0,0)$
$f(0,0) = 0$
3) Find the directional derivative of $\phi\,(x,y,z) = xy^2 + yz^3$ at the point (2,-1, 1) in the direction of the vector $i + 2j + 2k$.

Q.3 A) Attempt any two of the following question. 08

1) If $y = e^{ax}\cos(bx + c)$ then prove that
$y_n = r^n e^{ax}\cos(bx + c + n\phi)$ where $r = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}\frac{b}{a}$

2) If $z(x+y) = x^2 + y^2$ show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

3) Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$

B) Attempt any one of the following questions. **06**

1) State and prove L' Hospital's rule.

2) Verify Euler's theorem for the function

$$u = \sin^{-1}\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$$

Q.4 A) Attempt any two of the following questions. **10**

1) If $\bar{r}$ is the position vector of the point (x, y, z) and r is the modulus of $\bar{r}$ then prove that curl $r^n \bar{r} = \bar{0}$ and div $(r^n \bar{r}) = (n+3) r^n$

2) If $In = \frac{d^n}{dx^n}(x^n \log x)$ prove that, $\mathrm{In} = nI_{n-1} + (n-1)!$ hence, deduce that

$$I_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

3) Find the integral

$$\int_0^{\pi} x \sin^4 x \cos^6 x \, dx$$

B) Attempt any one of the following questions. **04**

1) If $x = r\cos\theta,\ y = r\sin\theta,$ prove that

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \quad \text{for} \quad x \neq 0, y \neq 0$$

2) Prove that grad Q is a vector normal to the surface $Q(x, y, z) = C$

Q.5 Attempt any two of the following questions. **14**

a) State and prove Leibnitz's theorem.

b) If $Z = f(x, y)$ is a function possessing continuous first order partial derivatives and $x = h(t),\ y = g(t)$ possessing continuous first order partical derivatives then prove that

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

c) 1) Evaluate the integral

$$\int_0^2 (4 - x^2)^{7/2} dx$$

2) If $\phi = x^2 + y^2 + z^2$, $\Psi = x^2y^2 + y^2z^2 + z^2x^2$, find $\nabla[\nabla\phi . \nabla\Psi]$.